

# Students opening up dimensions of variation for learning negative numbers

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**World Association of the Lesson studies**  
**International conference 2007, Hong Kong**



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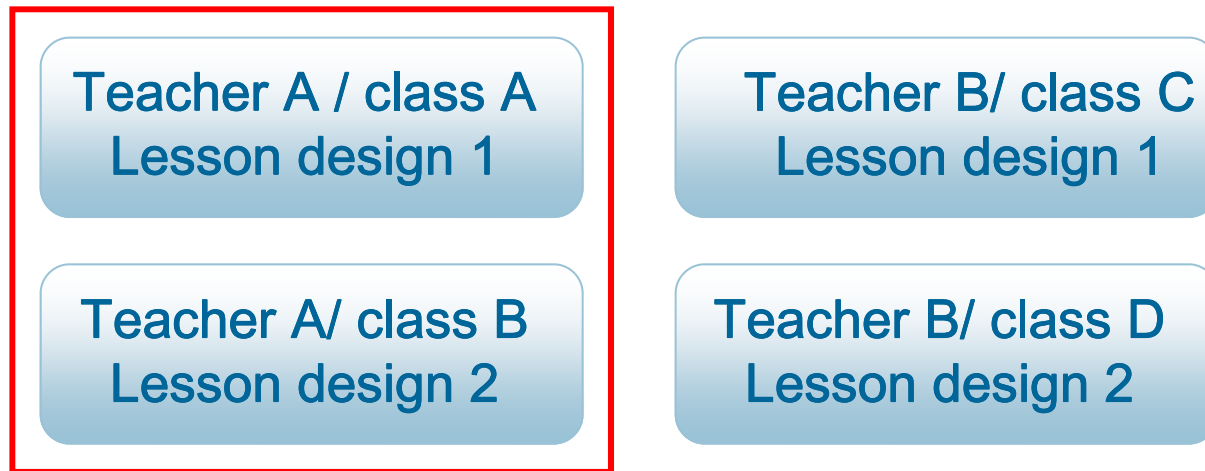
# Overview of the presentation

- Short background, research questions, framework, design, data
- "Two Lesson designs"
- Analysis and videoclips
- Conclusions

# The Design study

## ("The second step", after Learning study)

Two teachers conducted two different lesson designs in 7th grade (13 year old students, in the same school).



I will show that, in this study, lesson design 1 became similar to lesson design 2 in terms of critical features brought up during the lesson.



# Theoretical framework

- Variation theory (Marton & Tsui, 2004)
  - Analytic tool
  - Theory of learning
    - Learning is to differentiate (Gibson & Gibson, 1955)
    - Learning is to discern critical features of an object of learning
    - To be able to discern a feature the feature must vary and other features be invariant

# Data

- Pre- and post-tests
- Teachers collaborative work, the planning of the lessons (video recorded)
- Lessons (video recorded)
- Students' written working material



# Research questions

- How do the two lessons compare and what is possible to learn?
- What matters for students' learning about addition and subtraction of negative numbers?

# Lesson design

- It is a frame of critical features that is needed to be brought out by the teacher together with the students, to be discerned by the student.

In this study the lesson designs are given by the researcher and implemented by the teachers.

# The object of learning

- To understand addition and subtraction with negative numbers

$$(-5) - (-3) =$$





# Previous research about teaching and learning negative numbers

- Ball, D.L. (1993)
- Gallardo, A. (1995)
- Vlassis, J. (2004)

“The importance of noticing the difference between the signs (for subtraction and a negative number)”

# Critical features for learning addition and subtraction of negative numbers?

1. The difference between the signs  $(-5) - (-2) = (-3)$   
↑ ↑
2. Subtraction as a difference (not "take away")
3.  $a - b$  is not  $b - a$   $5 - 3 = 2$  but  $3 - 5 = (-2)$ , "it is always from the first position"  
↑ ↑
4. The numbersystem, "bigger numbers on the right side of the number line",  $-18$  is a smaller number than  $-3$  and  $1$

(The analysis of the critical aspects was made together with the participating teachers in the Learning study; Tuula Maunula, Joakim Magnusson and others)



# The lessons L1 and L2

Negative numbers as "debts"

Addition: Shared economy

$$\begin{array}{l} \text{Lisa} \quad \text{Tina} \\ (-3) + 5 = 2 \\ 5 + (-3) = 2 \end{array}$$

Subtraction: Compared economy

$$\begin{array}{l} \text{Lisa} \quad \text{Tina} \\ (-3) - 5 = (-8) \\ 5 - (-3) = 8 \end{array}$$

# Planned lesson designs

## Critical features

**Lesson 1**  
**(two)**

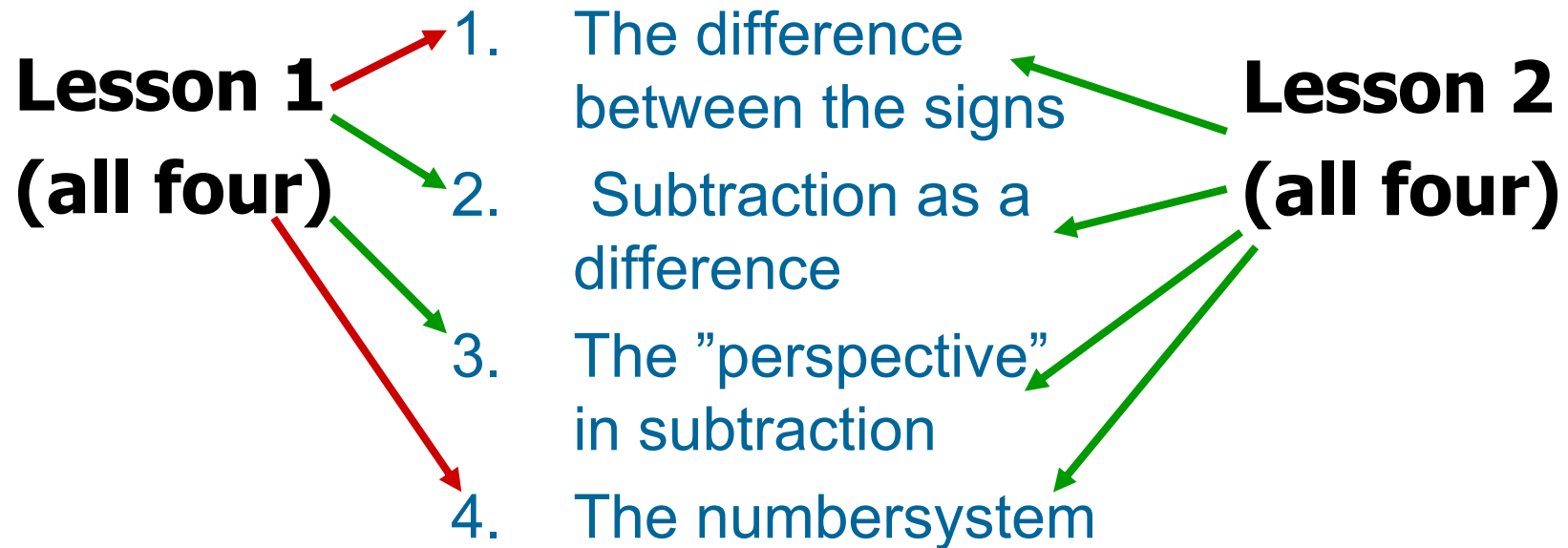
1. The difference between the signs
2. Subtraction as a difference
3. The "perspective" in subtraction
4. The numbersystem

**Lesson 2**  
**(four)**

Green: Intended to be brought up in the lesson

# Analysis of the lessons

## Critical features



**Red: Critical feature brought up in the lesson, not intended**

**Green: Intended to be brought up in the lesson**



# Lesson 1: The number system

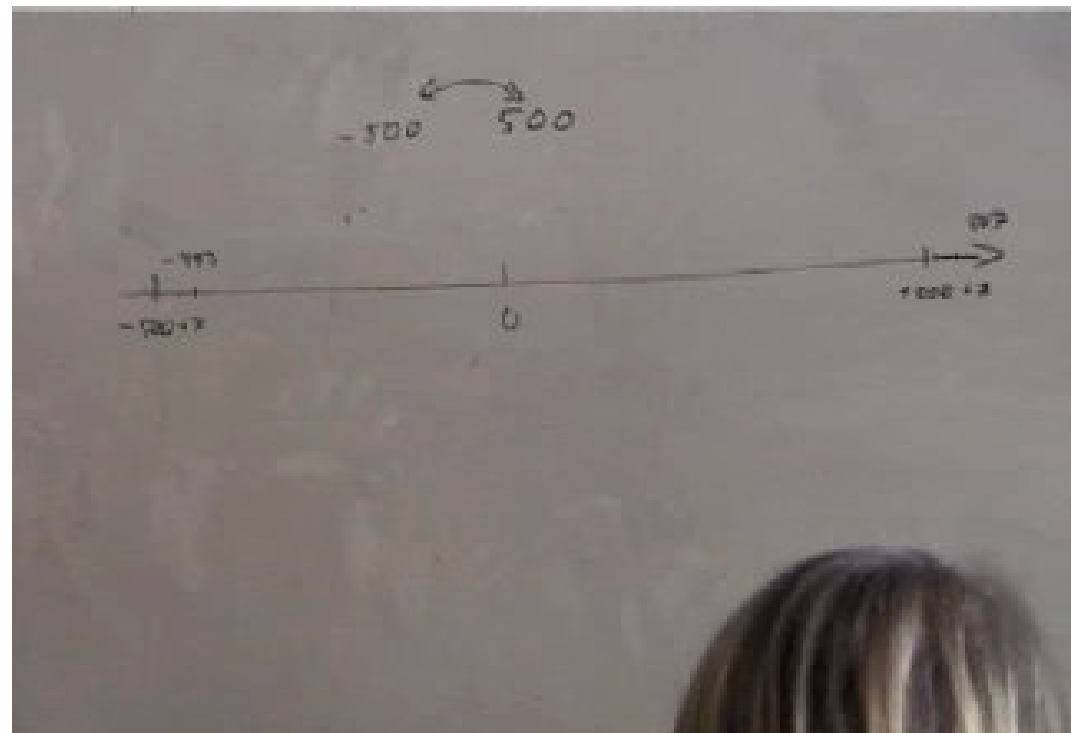
When you add you move towards the right side (aspect 4)

Teacher: Is there a difference between these numbers, we settle with these (-500 and 500) or is it the same number?

Student: If it were the same number it would be the same.

$$500 + 7 = 507$$

$$-500 + 7 = -493$$



# The same critical features, but in different ways...

## Lesson 1

**The number system** (Is 500 and -500 the same number? Adding the same number to a positive and a negative number)

**Differences between the signs** and between addition and subtraction

**The perspective in subtraction** (Is  $5-3$  the same as  $3-5$ ?)

**Subtraction seen as a difference** between two numbers and the difference between 'absolute difference' and the answer)

## Lesson 2

**The number system** (Is  $-3$  a bigger number than  $-5$ ?, comparing numbers)

**Difference between the signs** (Is the sign in  $5-3$  the same as  $-3$ ?)

**The perspective in subtraction** (Is  $5-3$  the same as  $3-5$ ?)

**Subtraction seen as a difference** between two numbers



# Results on tests

Table 2. Overall results on pre-and post-test.

<i>n</i>	<i>Lesson 1</i>		<i>Lesson 2</i>	
	<i>n=11</i>		<i>n=12</i>	
Task	Pre-test	Post-test	Pre-test	Post-test
Overall result	28%	58%	30%	65%
<b>Change</b>		+30%		+35%



# Conclusions

- A student's questions made it possible for critical features to be brought up during lesson 1 and it opened up dimensions of variation for learning Negative numbers
- This had an impact on what it was possible to learn during lesson 1 and therefore lesson 1 and 2 became similar even in terms of learning outcomes



# Thank you!

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# Dimension of variation (d.o.v.)

- A d.o.v is "what could vary" regarding the object of learning  
e.g. (a cup) color  
(not necessary the critical features, but it could be)

"Range of change" (Watson & Mason)  
e.g. red, yellow, white.....